A NOVEL APPROACH TO ADAPTIVE CONTROL OF NETWORKED SYSTEMS

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ABSTRACT

The insertion of communication network in the feedback adaptive control loops makes the analysis and design of networked control systems more and more complex. This paper addresses the stability problem of linear time-invariant adaptive networked control systems. Our approach is novel in that the knowledge of the exact values of all system parameters is not required. The case of state feedback is treated in which an upper bound on the norm of matrix *A* is required to be known. The priori knowledge of upper bound on norm *A* is not required in constructing the controller but it is required only to determine an upper bound on the transmission period *h* that guarantees the stability of the overall adaptive networked control system under an ideal transmission process, i.e. no transmission delay or packet dropout Rigorous mathematical proofs are established relies heavily on Lyapunov's stability criterion. Simulation results are given to illustrate the efficacy of our design approach.

Keywords: Networked control systems, Transmission period**,** Adaptive control, Lyapunov's stability**.**

1 INTRODUCTION

Networked control systems (NCSs) are feedback control systems with network communication channels used for the communications between spatially distributed system components like sensors, actuators and controllers. In recent years, the discipline of networked control systems has become a highly active research field. The use of networks as media to interconnect the different components in an industrial control system is rapidly increasing. For example in large scale plants and in geographically distributed systems, where the number and/or location of different subsystems to control make the use of single wires to interconnect the control system prohibitively expensive [1]. The primary advantages of an NCS are reduced system wiring, ease of system diagnosis

and maintenance, and increase system agility [2]. The insertion of the data network in the feedback control loop makes the analysis and design of an NCS more and more complex, especially for adaptive systems in which systems parameters not completely known. Conventional control theories with many ideal assumptions, such as synchronized control and non-delayed sensing and actuation, must be reevaluated before they can be applied to NCSs. Specifically; the following issues need to be addressed. The first issue is the network induced delay (sensor-to-controller delay and controller-to-actuator delay) that occurs while exchanging data among devices connected to the shared medium. This delay, either constant (up to jitter) or time varying, can degrade the performance of control systems designed without considering the delay and can even destabilize the system. Next, the network can be viewed as a web of unreliable transmission paths. Some packets not only suffer transmission delay but, even worse, can be lost during transmission [3].

The main challenge to be addressed when considering a networked control system is the stability of the overall NCSs. In this paper, we treat the stability analysis of networked control adaptive systems, when the network is inserted only between sensors and the controller. Under an ideal transmission process, i.e. no transmission delay or packet dropout, we have derived a sufficient condition on the transmission period that guarantees the NCS will be stable. This case is treated in [4] and [5], with completely known systems.

This paper is organized as follows; the problem is formulated in Section 2. The main result is given in Section 3. Section 4 presents an example and simulation results, finally we present our conclusions in Section 5.

2 FORMULATION OF THE PROBLEM

Consider an NCS shown in Fig. 1, in which sensor is clock-driven and both controller and actuator are event driven.

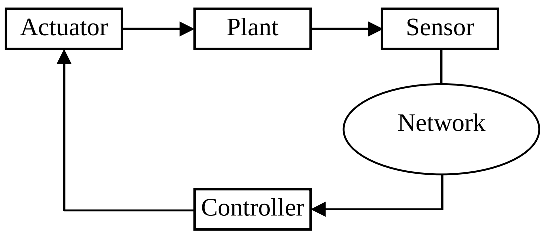

Figure 1 The block diagram of NCS

In Fig. 1, a class of linear time-invariant plants is described as

$$\dot{x}(t) = Ax(t) + bu(t) \quad t \in [t_k, t_{k+1}) \quad , k = 0,1, 2, ... \tag{1}$$

where $x(t)\in R^n$ is a state vector, $u(t_k)\in R$ is a control input vector, (A, b) is controllable, A is a constant matrix with *unknown elements*, b is a known constant vector. We assume that the control is updated at the instant t_k and kept constant until next control update is received at time t_{k+1}. Let h be the transmission period between successive transmissions, that is, $h = t_{k+1} - t_k$. For this paper, we assume that the transmission process is ideal, there are no delays, no data losses (packet losses) during the transmission. In future work, we will relax these assumptions.

Our objective is to design an adaptive stabilizer for the networked system and to find an upper bound on the time transmission period (sampling period) h such that the NCS is still stable.

The control input is of the form

$$u(t) = k^T(t)x(t_k) \tag{2}$$

where $k(t)$ is an n-dimensional *control parameter* vector, T denotes transpose. From Eqs. (1) and (2), we get

$$\begin{aligned}\dot{x}(t) &= Ax(t) + bk^T(t)x(t_k)\\ &= \bar{A}x(t) - bk^{*T}x(t) + bk^T(t)x(t_k)\end{aligned} \tag{3}$$

where $\bar{A} = A + bk^{*T}$ is Hurwitz matrix satisfying that $\bar{A}^T P + P\bar{A} = -Q$, P and Q are symmetric and positive–definite matrices, and k^* is the true value of $k(t)$. Define $\phi(t) = k(t) - k^*$ as the control parameter error vector, and $e(t) = x(t) - x(t_k)$ as the transmission error, Eq. (3) can be rewritten as

$$\dot{x}(t) = \bar{A}x(t) + b\phi^T(t)x(t_k) - bk^{*T}e(t) \tag{4}$$

3 MAIN RESULT

The main result of this paper will be treated in the following theorem.

Theorem 1: Let an NCS with linear time-invariant plant (1), an adaptive stabilizer with control input (2) is globally stable if the adaptive control law takes the form [6]

$$\dot{\phi}(t) = -\alpha\, x(t_k)x^T(t_k)Pb \tag{5}$$

and the transmission period satisfies $h < \min\{1, h_1, h_2, h_3\}$, where, α is an $n\times n$ symmetric positive-definite adaptation gain matrix, and

$$h_1 = \frac{1}{A_{upp}}\ln\left(1 + \frac{A_{upp}}{\zeta_1}\right)$$

$$h_2 = \frac{1}{A_{upp}}\ln\left(1 + \frac{\beta\lambda_{\min}(Q)A_{upp}}{\zeta_2}\right)$$

$$h_3 = \frac{1}{A_{upp}}\ln\left(1 + \frac{\left((1-\frac{\beta}{4}) - \sqrt{(1-\frac{\beta}{4})^2 - (1-\beta)}\right)A_{upp}}{\zeta_1}\right)$$

where $\zeta_1 = A_{upp} + \|bk^T(t_k)\| + \frac{1}{2}\|b\|^2\|p\|\|x(t_k)\|^2\|\alpha\|$

and $\zeta_2 = 4\zeta_1\|P\|\left(1 + \lambda_{\min}(Q) + \|\bar{A}\| + A_{upp} + \|bk^T(t)\|\right)$.

To prove the stability of the NCS, firstly, we will find an upper bound on the transmission error $e(t)$, a lower and an upper bound on the state $x(t)$, and finally, we will use these bounds in Lyapunov function to prove Theorem 1.

Lemma 1: (Transmission Error Upper Bound) The transmission error $e(t)$ is bounded between two successive transmissions by

$$\|e(t)\| \le \gamma\|x(t_k)\| \tag{6}$$

where

$$\gamma = \frac{A_{upp} + \|bk^T(t_k)\| + \frac{1}{2}\|b\|^2\|p\|\|x(t_k)\|^2\|\alpha\|}{A_{upp}}\left(e^{A_{upp}(t-t_k)} - 1\right),$$

A_{upp} is an upper bound on A such that; $\|A\| \leq A_{upp}$.

Proof: From the definition of $e(t)$, it can be found that

$$\dot{e}(t) = \dot{x}(t) = Ax(t) + bk^T(t)x(t_k)$$
$$= Ae(t) + Ax(t_k) + bk^T(t)x(t_k)$$

Taking the integral on both sides, and taking into account that $e(t_k) = 0$, we have

$$e(t) = e(t_k) + \int_{t_k}^{t} \left(Ae(s) + Ax(t_k) + bk^T(t)x(t_k)\right)ds$$
$$= [Ax(t_k) + bk^T(t_k)x(t_k)](t - t_k)$$
$$- \frac{1}{2} bb^T p\, x(t_k)x^T(t_k)\alpha x(t_k)(t - t_k)^2$$
$$+ \int_{t_k}^{t} Ae(s)ds$$

If we choose t-t_k < 1, Therefore,

$$\|e(t)\| \leq [\|A\|\|x(t_k)\| + \|bk^T(t_k)\|\|x(t_k)\|$$
$$+ \frac{1}{2}\|b\|^2\|p\|\|x(t_k)\|^3\|\alpha\|](t - t_k)$$
$$+ \int_{t_k}^{t} \|A\|\|e(s)\|ds$$

If we know an upper bound of A that is; $\|A\| \leq A_{upp}$, and applying Bellman-Gronwall Lemma [2], yields

$$\|e(t)\| \leq \int_{t_k}^{t} [A_{upp} + \|bk^T(t_k)\| + \frac{1}{2}\|b\|^2\|p\|\|x(t_k)\|^2\|\alpha\|]$$
$$\times \|x(t_k)\| \exp\left(\int_{s}^{t} A_{upp}\, dw\right)ds$$

Then

$$\|e(t)\| \leq \gamma\|x(t_k)\|$$

Lemma 2: The state of the NCS, x(t), between successive transmissions is bounded by

$$(1 - \gamma)\|x(t_k)\| \leq \|x(t)\| \leq (1 + \gamma)\|x(t_k)\| \qquad (7)$$

Proof: As $e(t) = x(t) - x(t_k)$, then

$$\|x(t_k)\| - \|e(t)\| \leq \|x(t)\| \leq \|e(t)\| + \|x(t_k)\|$$

Using Eq. (6), it can be concluded that

$$(1 - \gamma)\|x(t_k)\| \leq \|x(t)\| \leq (1 + \gamma)\|x(t_k)\|$$

Now we turn our attention to proof of Theorem 1. Consider a positive-definite Lyapunov function $V(t)$ of the form

$$V(t) = x^T(t)Px(t) + \phi^T(t)\alpha^{-1}\phi(t) \qquad (8)$$

Differentiating $V(t)$ with respect to t, we have

$$\dot{V}(t) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + \dot{\phi}^T(t)\alpha^{-1}\phi(t) + \phi^T(t)\alpha^{-1}\dot{\phi}(t) \qquad (9)$$

Substituting for $\dot{x}(t)$ and $\dot{\phi}(t)$ from Eqs. (4) and (5), there results

$$\begin{aligned}\dot{V}(t) = {} & x^T(t)\bar{A}Px(t) + x^T(t_k)\phi(t)b^TPx(t) \\ & - e^T(t)k^*b^TPx(t) + x^T(t)P\bar{A}x(t) \\ & + x^T(t)Pb\phi^T(t)x(t_k) \\ & - x^T(t)Pbk^{*T}e(t) \\ & - b^TPx(t_k)x^T(t_k)\phi(t) \\ & - \phi^T(t)x(t_k)x^T(t_k)Pb\end{aligned} \qquad (10)$$

Rearranging Eq. (10), yields

$$\begin{aligned}\dot{V}(t) = {} & -x^T(t)Qx(t) + 2x^T(t)Pb\phi^T(t)x(t_k) \\ & - 2x^T(t)Pbk^{*T}e(t) \\ & - 2x^T(t_k)Pb\phi^T(t)x(t_k)\end{aligned} \qquad (11)$$

$\dot{V}(t)$ becomes bounded from above as

$$\begin{aligned}\dot{V}(t) \leq {} & -\lambda_{\min}(Q)\|x(t)\|^2 \\ & + 2\|P\|\|b\phi^T(t)\|\|e(t)\|\|x(t_k)\| \\ & + 2\|P\|\|bk^{*T}\|\|x(t)\|\|e(t)\|\end{aligned} \qquad (12)$$

From (6) and (7), where we choose h < 1, then

$$\begin{aligned}\dot{V}(t) \leq {} & -\lambda_{\min}(Q)\|x(t)\|^2 \\ & + \frac{2\gamma}{(1-\gamma)}\|P\|\|b\phi^T(t)\|\|x(t)\|\|x(t_k)\| \\ & + 2\gamma\|P\|\|bk^{*T}\|\|x(t)\|\|x(t_k)\|\end{aligned} \qquad (13)$$

Using (7), and rearranging,

$$\dot{V}(t) \le \frac{1}{(1-\gamma)}\|x(t)\| \Big(-(1-\gamma)^2 \lambda_{\min}(Q) + 2\gamma\|P\|\|bk^T(t) - bk^{*T}\| + 2\gamma(1-\gamma)\|P\|\|bk^{*T}\|\Big)\|x(t_k)\| \quad (14)$$

By choosing $\gamma < 1$ to guarantee that $(1-\gamma) > 0$, we can conclude that $h < h_1$, where

$$h_1 = \frac{1}{A_{upp}} \ln\left(1 + \frac{A_{upp}}{\zeta_1}\right) \quad (15)$$

Using, $\|bk^{*T}\| \le \|\overline{A}\| + A_{upp}$, then $\dot{V}(t)$ becomes

$$\dot{V}(t) \le \frac{1}{(1-\gamma)}\|x(t)\| \Big(-(1-\gamma)^2 \lambda_{\min}(Q) + 2\gamma\|P\|\left\{\|bk^T(t)\| + \|\overline{A}\| + A_{upp}\right\} + 2\gamma(1-\gamma)\|P\|\left\{\|\overline{A}\| + A_{upp}\right\}\Big)\|x(t_k)\| \quad (16)$$

Again, by choosing

$$\gamma < \frac{\beta\lambda_{\min}(Q)}{(4\|P\|)\left(1 + \lambda_{\min}(Q) + \|\overline{A}\| + A_{upp} + \|bk^T(t)\|\right)},$$

and

$0 < \beta < 1$, we have $h < h_2$, where

$$h_2 = \frac{1}{A_{upp}} \ln\left(1 + \frac{\beta\lambda_{\min}(Q)A_{upp}}{\zeta_2}\right) \quad (17)$$

Substituting for γ in (16), we get

$$\dot{V}(t) \le \frac{\lambda_{\min}(Q)}{(1-\gamma)}\|x(t)\|\left(-(1-\gamma)^2 + \beta - \frac{\beta}{2}\gamma\right)\|x(t_k)\| \quad (18)$$

Finally, by choosing

$\gamma < (1-\frac{\beta}{4}) - \sqrt{(1-\frac{\beta}{4})^2 - (1-\beta)}$, we have $h < h_3$, where

$$h_3 = \frac{1}{A_{upp}} \ln\left(1 + \frac{\left((1-\frac{\beta}{4}) - \sqrt{(1-\frac{\beta}{4})^2 - (1-\beta)}\right)A_{upp}}{\zeta_1}\right) \quad (19)$$

and, we can conclude that $\dot{V}(t) < 0$, if h satisfies $h < \min\{1, h_1, h_2, h_3\}$ defined in Eqs. (15), (17), and (19), respectively. Therefore, $x(t)$, $\phi(t)$, and $V(t)$ are bounded for all $t \ge t_0$ and the over all system is globally stable.

4 SIMULATION RESULTS

Now, we demonstrate the applicability of our approach through the following example. Consider the plant parameters

$$A = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Assume the desired plant parameters

$$\overline{A} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

Let

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

Assume A is *unknown* but only A_{upp} is known (take $A_{upp} = 3$).

Figure 2 shows the simulation results for the networked control system with $x(0) = [1\ 1]^T$, $\alpha = I$ (identity matrix), $\beta = 0.9$, $k(0) = [0\ 0]^T$, it is found that $h_1 < 0.1729$s, $h_2 < 0.0125$s, and $h_3 < 0.0149$s. Before starting simulation we know that, $h < 0.0125$s, but with simulation proceeds, h can be found on-line as shown in Fig. 3 (we take $h = 0.002$s). Figure 4 shows the simulation results for the networked control system with $x(0) = [1\ 1]^T$, $\alpha = I$, $\beta = 0.9$, $k(0) = [1\ 1]^T$, it is found that $h_1 < 0.1279$s, $h_2 < 0.0069$s, and $h_3 < 0.0104$s. Before starting simulation we know that, $h < 0.0069$s (we take $h = 0.001$s), also with simulation proceeds, h can be found on-line as shown in Fig. 5. From Eqs. (15), (17), (19) and Figs. (3), (6), it can be concluded that h_3 is the minimum transmission period.

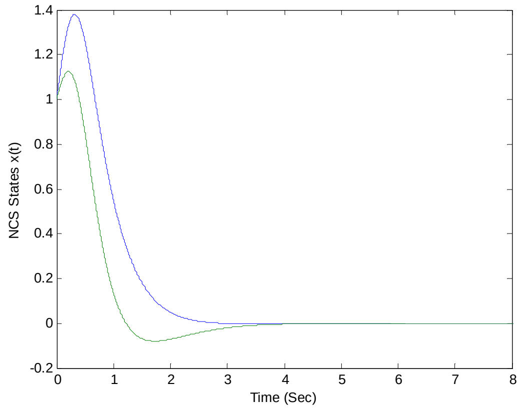

Figure 2 NCS states $x(t)$

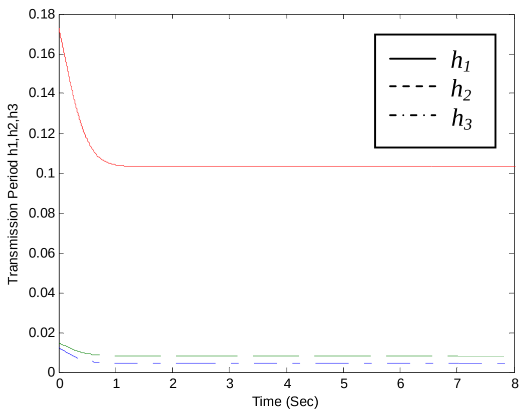

Figure 3 Transmission period h

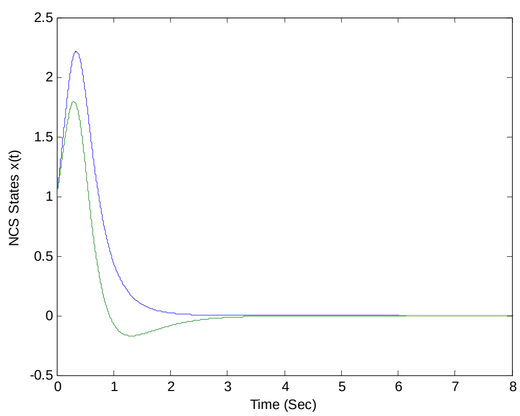

Figure 4 NCS states $x(t)$

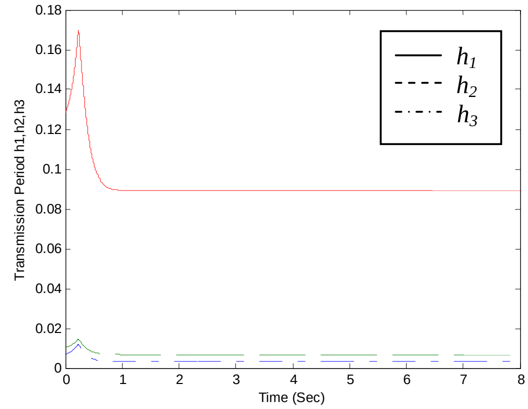

Figure 5 Transmission period h

1 CONCLUSIONS

The paper addresses the stability analysis of linear time-invariant adaptive networked control systems. The case of state feedback is in which only an upper bound on the norm of matrix A is required. As shown in theorem 1, the priori knowledge of upper bound on norm A is not required in constructing the controller but it is required only to determine an upper bound on the transmission period h that guarantees the stability of the overall adaptive networked control system under an ideal transmission process, i.e. no transmission delay or packet dropout. In future work we will try to relax these assumptions. Rigorous mathematical proofs are established relies heavily on Lyapunov's stability criterion. Simulation results are given to illustrate the efficacy of our design approach. It is verified that, if the sampling period of the network is less than the upper bound on h, the control parameters of the adaptive controller are bounded and that the NCS states converge to zero as time tends to infinity value as time evolves.

ACKNOWLEDGEMENTS

This work is supported by National Natural Science Foundation of China, Grant #60574088 and #60274014.

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