Solving Peak Power Problems in Orthogonal Frequency Division Multiplexing

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ABSTRACT

In this paper, the problem of reducing the peak to average power ratio (PAPR) in an Orthogonal frequency division multiplexing (OFDM) system is considered. We propose the use of Walsh Hadamard Transform (WHT) as an intelligent scaling factor to reduce the dynamic range of the OFDM signal without the risk of amplifying the noise when restoring the signal to its original level. This technique offers an excellent solution to all of peak power problems in OFDM systems and without any loss in terms of spectral efficiency and without any side information being transmitted, and can be applied with low computational complexity.

Keywords: OFDM, Walsh Hadamard Transform, HPA, PAPR.

1 INTRODUCTION

The demand for high data rate services has been increasing very rapidly and there is no slowdown in sight. Often, these services require very reliable data transmission over very harsh environments. Most of these transmission systems experience many degradations, such as large attenuation, noise, multi-path, interference, time variation, nonlinearities, Doppler spread and must meet many constraints, such as finite transmit power and efficient bandwidth. One physical layer technique that has recently gained much popularity due to its robustness in dealing with these impairments is multi-carrier modulation, specially, OFDM.

Unfortunately, one particular major problem with OFDM signals that is often cited as the major drawback of multi-carrier transmissions is its large envelope fluctuation, which is quantified by the parameter called peak-to-average power ratio (PAPR). Since most practical transmission systems are peak-power limited, designing the system to operate in a perfectly linear region often implies operating at power levels well below the maximum power available.

In practice, to avoid operating the amplifier with extremely large back-offs occasional saturation of the amplifiers or clipping in the digital-to-analog converters must be allowed. This additional process is a nonlinear process which creates inter-modulation distortion that increases the bit-error-rate (BER) in standard linear receivers, and also causes spectral widening of the transmit signal that increases adjacent channel interference to the other users.

A number of approaches have been studied to deal with the PAPR problem in OFDM, including amplitude clipping, filtering, coding, tone reservation, tone injection ,active constellation extension, and multiple signal representation techniques such as partial transmit sequence, selected mapping. These techniques achieve PAPR reduction at the expense of transmits signal power increase, bit error rate increase, data rate loss, computational complexity increase, and so on. [2]

In this paper, we will introduce the use of Walsh Hadamard Transform (WHT) to solve the peak power problem in OFDM systems. The novelty of this paper is mainly in documenting the performance of the proposed method in different scenarios related to OFDM transmission. Specifically, improvements in BER performance, recovery from constellation warping and the reduction of out-of-band spectral regrowth in the channels are documented through computer simulations.

2 PEAK-TO-AVERAGE POWER RATIO (PAPR)

One particular problem with OFDM signals, which is often cited as its major drawback is its large envelope fluctuations, which is usually measured by parameter called PAPR. PAPR is defined as the ratio of the peak instantaneous power and the average power i.e., mathematically expressed as:

$$PAPR = \frac{\max|s(t)|^2}{mean|s(t)|^2}, \; 0 < t < T_s \qquad (1)$$

Where s(t) is OFDM symbol with interval 0<t<Ts.

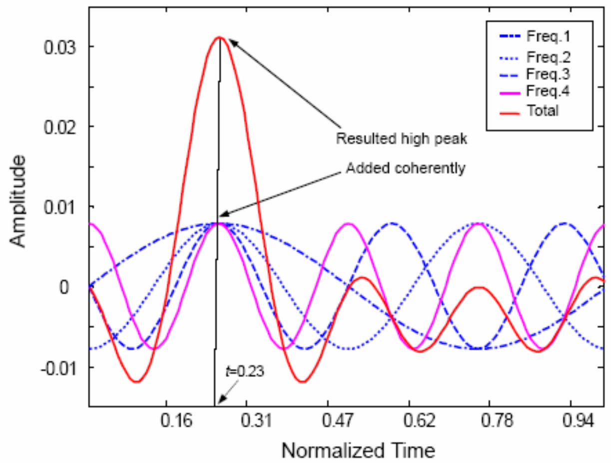


Fig. (1) Coherent superposition of Sub-carriers

Fig. 1 illustrates how a coherent superposition among modulated carrier results a high peak. Signal with frequency Freq.1 (the lowest), Freq.2, Freq.3 and Freq.4 (the highest) have the same peak and phase at time t=0.23. OFDM signal is obtained from the summation of these signals which results in a new signal with high peak of approximately four times compared to individual signal. PAPR is then calculated from the ratio of this instantaneous peak and average power of this signal.

Eq. (1) means that this PAPR is evaluated per OFDM symbols, and thus there is a need to study the PAPR statistical performance of OFDM systems. The cumulative distribution function (CDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. In the literature, the complementary CDF (CCDF) or survivor function is commonly used instead of the CDF itself. The CCDF of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold. The survivor function will be one of the performance enhancement measures of the proposed power problem solving technique.

3 WALSH HADAMARD TRANSFORM

The Walsh-Hadamard Transform (WHT) is perhaps the best known of the non sinusoidal orthogonal transforms. The Walsh-Hadamard transform of a signal x, of size N = 2n, is the matrix-vector product WHTN ·x, [5] where

$$WHT_N = \bigotimes_{i=1}^{n} DFT_2 = DFT_2 \otimes \overset{n}{\ldots} \otimes DFT_2$$

And $DFT_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ is the 2-point DFT matrix, and $\otimes$ denotes the tensor or Kronecker product. The tensor product of two matrices is obtained by replacing each entry of the first matrix by that element multiplied by the second matrix. Thus, for example,

$$WHT_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

The WHT has gained prominence in digital signal processing applications, since it can be computed using additions and subtractions only consequently, its hardware implementation is simpler.

3.1 Fast Walsh-Hadamard Transform (FWHT)

As the FFT is an algorithm to compute the DFT efficiently, similarly the FWHT is an algorithm to compute the WHT efficiently. The FWHT can be expressed as: [4]

$$W(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)g(x,u) \qquad (2)$$

Where $g(x,u) = [\prod_{i=0}^{n-1} (-1)^{b_i(x)b_{n-1-i}(u)}]$

The FWHT can be derived using matrix factoring or matrix partitioning techniques. The signal flow graph for a 4 point FWHT is shown in Figure 2.

In a digital system the 1/4 multiplier can be simply implemented in two arithmetic shifts. The number of additions and subtractions needed to compute the four WHT coefficients is 4 x $\log_2 4$ = 8.

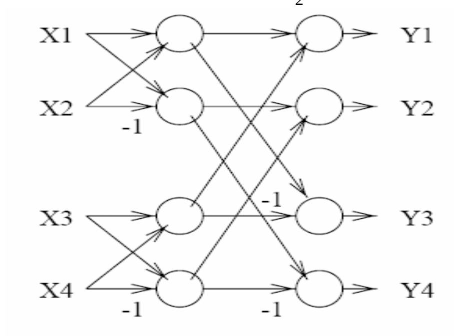


Fig. (2) 4 point FWHT

3.2 OFDM system using FWHT

We have examined the use of FWHT with OFDM with different lengths.

The idea: since the peak value of the OFDM symbol results at the instant of coherent addition of sub-carriers, it is possible to reduce this peak by modifying the sub-carriers initial phases so as to avoid this condition. This can be done via successive phase shifts, we have tried when summing up a number of orthogonal sinusoidal waveforms with successive phase shifts with different phase shift step in each time we try and we have got the optimum case at **phase shift step** = π which is equivalent to modifying the OFDM symbol representation to be as follows:

$$y_n = \sum_{k=0}^{N-1} (-1)^k X_k e^{j2\pi \frac{nk}{N}}$$

The above equation can be approximately implemented as shown below in figure (3) in the proposed block diagram of OFDM system.

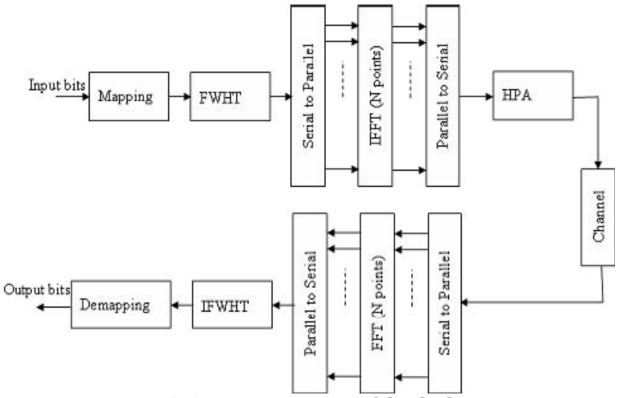


Fig.(3) OFDM system block diagram

In this paper, computer simulations are used to test the performance of the proposed method for solving the peak power problems of OFDM systems. In the simulations, 16- QAM modulation scheme is selected in OFDM with a total of 1024 sub-carriers.

4 POWER AMPLIFIER MODEL

A short description of power amplifier models will be given in this section. Consider an input signal in polar coordinates as $x = \rho e^{j\phi}$

The output of the power amplifier can be written as $g(x) = M(\rho)e^{j(\phi + P(\rho))}$

Where M(ρ) represents the AM/AM conversion and P(ρ) the AM/PM conversion characteristics of the power amplifier. Several models have been developed for nonlinear power amplifiers; in this paper we have applied the SSPA model with OFDM system. The conversion characteristics of solid-state power amplifier are modeled by Rapp's SSPA with characteristic:

$$v_{out} = \frac{v_{in}}{(1 + (|v_{in}|/v_{sat})^{2p})^{\frac{1}{2p}}} \quad (3)$$

Where v_{out} and v_{in} are complex i/p & o/p, v_{sat} is the output saturation level (v_{sat}=A), and P is "knee factor" that controls the smoothness of the transition from the linear region to the saturation region of characteristic curve (we use P=2). Since the above described model assume Class A operation of the HPA, and due to its poor efficiency that limits its applications for portable communication systems, it is better to modify the model to different classes of operation as follows:

First of all, it is well known that the amplifier output goes to maximum saturation value at certain input value, it is useful to determine the input value at which the output reaches its maximum value, and this is done as follows:

$$\frac{dv_{out}}{dv_{in}} = 0$$

$$\therefore \frac{(1 + (\frac{v_{in}}{v_{sat}})^2) - v_{in}(0 + \frac{2v_{in}}{v_{sat}^2})}{(1 + (\frac{v_{in}}{v_{sat}})^2)^2} = 0$$

$$1 + (\frac{v_{in}}{v_{sat}})^2 - 2(\frac{v_{in}}{v_{sat}})^2 = 0 \quad \therefore (\frac{v_{in}}{v_{sat}})^2 = 1$$

The above result denotes that the output reaches its maximum value $v_{out} = v_{sat}$ at $v_{in} = v_{sat}$, this implies that the ratio between the input and the saturation output is determined by the constant value in the de-numerator which is equal to one in the Raap's model.

Thus, Raap's SSPA model can be modified as follows:

$$v_{out} = \frac{v_{in}}{(x^2 + (|v_{in}|/v_{sat})^{2p})^{\frac{1}{2p}}}$$

Where x is the required ratio between the input and the saturation output, which determine the slope of the characteristic curve and consequently the Class of the amplifier, we have chosen x =1, 0.5-0.25, < 0.25 for Classes A, B and C respectively. As shown in figure (4) below.

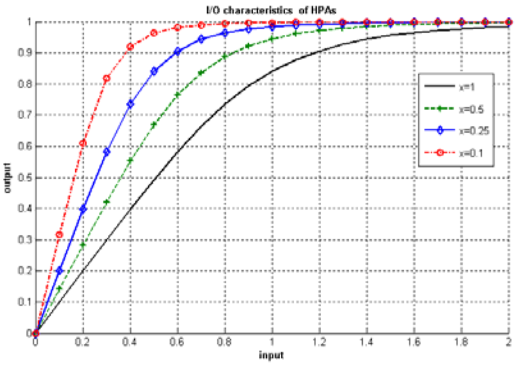
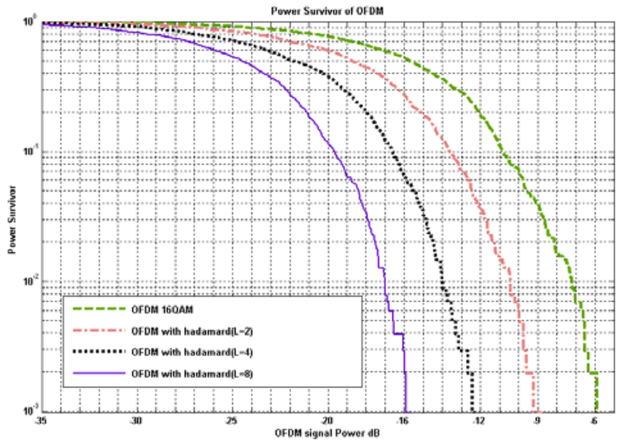


Figure (4) I/O characteristics of SSPA classes

5 SIMULATION RESULTS

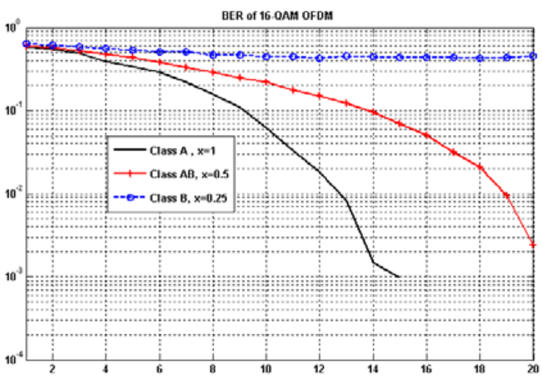
In this section we will discuss the benefits of using FWHT with OFDM system for solving the peak power related problems; we have simulated an OFDM system with 16-QAM and 1024 sub-carriers with and without FWHT, it is noticed that although there is no significant reduction in the PAPR value (about 1 dB) when using FWHT with OFDM system, the dynamic range is reduced in accordance with the size of FWHT. As depicted in figure (5) that shows the power survivor instead of the PAPR survivor. It is noticed that when using FWHT with size $\mathbf{N=2^n}$ the total power of the OFDM system is reduced by **3n dB,** which is a very good chance to perform an intelligent scaling that avoids the risk of amplifying the noise when restoring the signal to its original level at the receiver.



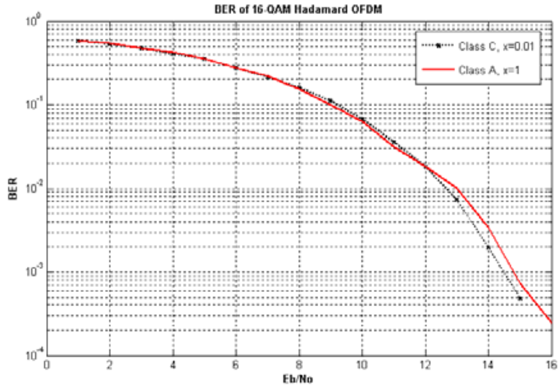
Fig(5) power survivor of OFDM with WHT

5.1 BER Performance Results

Bit error rate (BER) are a typical performance measure for quantifying the benefits of using FWHT with OFDM system to counteract the in-band distortions due to NLA. The BER performance as a function of the signal to noise ratio (SNR) Eb/No in OFDM systems with different classes HPA with and without FWHT are plotted in Fig.6 (a-b)



(a)without WHT



(b) with WHT

Fig (6) effect of NLA on BER performance: Figure

(6-a) demonstrates that in the cases considered, no reliable detection can be performed when applying either class B or C amplifiers. While figure (6-b) shows that When FWHT is used with OFDM, there will be approximately no non-linear effects even when applying class C amplifiers.

5.2 Constellation Warping Results

Non-linear distortions result in the generation of unwanted spectral energy both in-band and out of band as shown above. The in-band energy causes distortion of the transmitted signal through constellation warping and degrades the system BER performance. Figure (7) shows constellation warping results using the scatter pattern of the received signals in systems without and with FWHT, respectively, the interference is so great that the constellation points are severely blurred. The spreading of the constellation points is the result of in-band interference that evidences itself in a fashion similar to AWGN, although the latter is not present in the simulations in this sub-section.

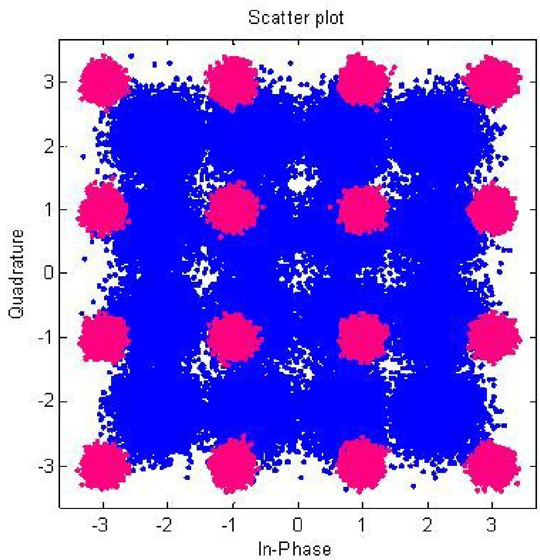


Fig. (7) Constellation wrapping

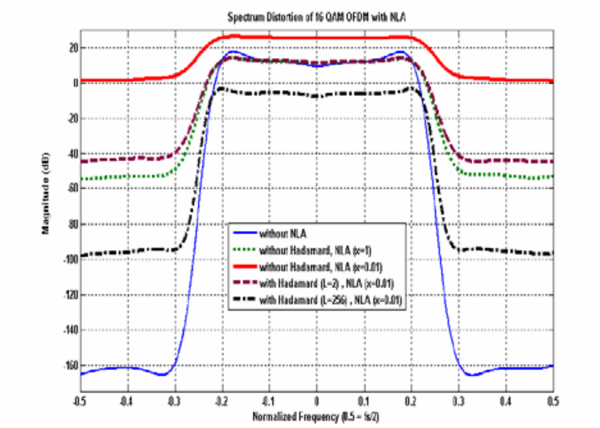


Fig (8) Spectrum Distortion due to NLA

In Figure (8), the spectrum spreading due to different classes of NLA with and without FWHT is shown; it is shown that when using FWHT, there is no significant spectrum distortion, in agreement with the BER performance results discussed above.

6 CONCLUSIONS

- The large dynamic range of OFDM signal result in Non-linear distortions as generation of unwanted spectral energy both in-band and out of band when subjected to Nonlinear elements. The in-band energy causes distortion of the transmitted signal through constellation warping and degrades the system BER performance.
- Walsh Hadamard Transform is used with OFDM systems as an intelligent scaling factor to reduce the dynamic range of the OFDM signal without the risk of amplifying the noise when restoring the signal to its original level. This technique offers an excellent solution to all of peak power problems in OFDM systems and without any loss in terms of spectral efficiency and without any side information being transmitted, and can be applied with low computational complexity.
- The use of WHT with OFDM system enables the use of the high efficiency Class C amplifier without affecting the BER performance.

REFERENCES

[1] Jian Li and Jacek Ilow, “Adaptive Volterra Predistorters for Compensation of Non-linear Effects with Memory in OFDM Transmitters”, Proceedings of the 4th Annual Communication Networks and Services Research Conference (CNSR’06), © 2006 IEEE.

[2] Le Hu, Chisheng Li, Guojin Wan, "A Modified Sub-optimal Algorithm for Reducing PAPR of Multicarrier Communication Systems” First International Conference on Innovative Computing, Information and Control - Volume I (ICICIC'06), 2006.

[3] M. Mehendale, S.D. Sherlekar, "Low Power Code Generation of Multiplication-free Linear Transforms", 12th International Conference on VLSI Design - 'VLSI for the Information Appliance', 1999.

[4] T. Acharya, “Image Processing Principals and applications”, John Willey, New Jersey, © 2005.

[5] M. Furis, P. Hitcenko, and J. R. Johnson, “Cache Miss Analysis of WHT Algorithms”, Proc. International Conference on Analysis of Algorithms, Discrete Mathematics and Theoretical Computer Science, 2005.