

# REDUCTION OF INTERCARRIER INTERFERENCE IN OFDM SYSTEMS

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## ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) is a promising technique for the broadband wireless communication system. However, a special problem in OFDM is its vulnerability to frequency offset errors due to which the orthogonality is destroyed that result in Intercarrier Interference (ICI). ICI causes power leakage among subcarriers thus degrading the system performance. This paper will investigate the effectiveness of Maximum-Likelihood Estimation (MLE), Extended Kalman Filtering (EKF) and Self-Cancellation (SC) technique for mitigation of ICI in OFDM systems. Numerical simulations of the ICI mitigation schemes will be performed and their performance will be evaluated and compared in terms of bit error rate (BER), bandwidth efficiency and computational complexity.

**Keywords:** Orthogonal Frequency Division Multiplexing (OFDM), Intercarrier Interference (ICI), Carrier Frequency Offset (CFO), Carrier to Interference Ratio (CIR), Maximum Likelihood (ML), Extended Kalman Filtering (EKF).

## 1. Introduction

Orthogonal frequency division multiplexing (OFDM), because of its resistance to multipath fading, has attracted increasing interest in recent years as a suitable modulation scheme for commercial high-speed broadband wireless communication systems. OFDM can provide large data rates with sufficient robustness to radio channel impairments. It is very easy to implement with the help of Fast Fourier Transform and Inverse Fast Fourier Transform for demodulation and modulation respectively [1].

It is a special case of multi-carrier modulation in which a large number of orthogonal, overlapping, narrow band sub-channels or subcarriers, transmitted in parallel, divide the available transmission bandwidth [2]. The separation of the subcarriers is theoretically minimal such that there is a very compact spectral utilization. These subcarriers have different frequencies and they are orthogonal to each other [3]. Since the bandwidth is narrower, each sub channel requires a longer symbol period. Due to the increased symbol duration, the ISI over each channel is reduced.

However, a major problem in OFDM is its vulnerability to frequency offset errors between the transmitted and received signals, which may be caused by Doppler shift in the channel or by the difference between the transmitter and receiver local oscillator frequencies [4]. In such situations, the orthogonality of the carriers is no longer maintained, which results in Intercarrier Interference (ICI). ICI results from the other sub-channels in the same data block of the same user. ICI problem would become more complicated when the multipath fading is present [5]. If ICI is not properly compensated it results in power leakage among the subcarriers, thus degrading the system performance.

In [6], ICI self-cancellation of the data-conversion method was proposed to cancel the ICI caused by frequency offset in the OFDM system. In [7], ICI self-cancellation of the data-conjugate method was proposed to minimize the ICI caused by frequency offset and it could reduce the peak average to power ratio (PAPR) than the data-conversion method. In [8], self ICI cancellation method which maps the data to be transmitted onto adjacent pairs of subcarriers has been described. But this method is less bandwidth efficient. In [9], the joint Maximum Likelihood symbol-time and carrier frequency offset (CFO) estimator in OFDM systems has been developed. In this paper, only carrier frequency offset (CFO) is estimated and is cancelled at the receiver. In addition, statistical approaches have also been explored to estimate and cancel ICI [10].

**Organization:** This paper is organized as follows: In section 2, the standard OFDM system has been described. In section 3, the ICI mitigation schemes such as Self-Cancellation (SC), Maximum Likelihood Estimation (MLE) and Extended Kalman Filtering (EKF) methods have been described. In section 4, simulations and results for the three methods has been shown and are compared in terms of bandwidth efficiency, bit error rate (BER) performance. Section 5 concludes the paper and inference has been given.

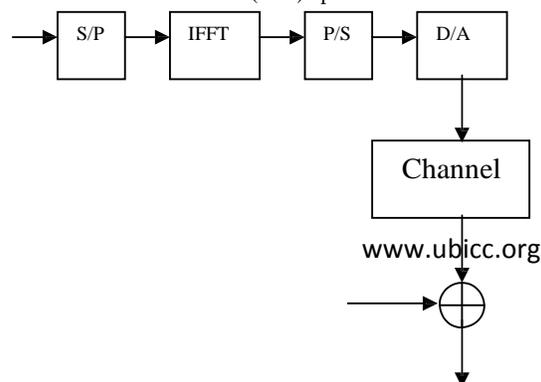
## 2. System Description

The block diagram of standard OFDM system is given in figure 1. In an OFDM system, the input data stream is converted into N parallel data streams each with symbol period  $T_s$  through a serial-to-parallel Port. When the parallel symbol streams are generated, each stream would be modulated and carried over at different center frequencies. The sub-carriers are spaced by  $1/NT_s$  in frequency, thus they are orthogonal over the interval  $(0, T_s)$ . Then, the N symbols are mapped to bins of an inverse fast Fourier transform (IFFT). These IFFT [11] bins correspond to the orthogonal sub-carriers in the OFDM symbol. Therefore, the OFDM symbol can be expressed as

$$x(n) = \frac{1}{N} \sum_{m=0}^{N-1} X_m e^{j2\pi nm/N} \quad (1)$$

where the  $X_m$ 's are the base band symbols on each sub-carrier. The digital-to-analog (D/A) converter then creates an analog time-domain signal which is transmitted through the channel.

At the receiver, the signal is converted back to a discrete N point sequence  $y(n)$ , corresponding to each sub-carrier. This discrete signal is demodulated using an N-point Fast Fourier Transform (FFT) operation at the receiver.



**w(n)**

**Figure 1: OFDM System Model**

The demodulated symbol stream is given by:

$$Y(m) = \sum_{n=0}^{N-1} y(n)e^{-j2\pi nm/N} + w(m) \quad (2)$$

where  $w(m)$  corresponds to the FFT of the samples of  $w(n)$ , which is the Additive White Gaussian Noise (AWGN) introduced in the channel.

### 3. ICI Mitigation Schemes

#### 3.1 Self-Cancellation (SC) Scheme

In this scheme, data is mapped onto group of subcarriers with predefined coefficients. This results in cancellation of the component of ICI within that group due to the linear variation in weighting coefficients, hence the name self-cancellation. The complex ICI coefficients  $S(l-k)$  are given by

$$S(l-k) = \frac{\sin(\pi(l+\varepsilon-k))}{N\sin(\pi(l+\varepsilon-k)/N)} \exp(j\pi(1-1/N)(l+\varepsilon-k)) \quad (3)$$

#### 3.1.1 ICI Canceling Modulation

The ICI self-cancellation scheme requires that the transmitted signals be constrained such that  $X(1) = -X(0)$ ,  $X(3) = -X(2) \dots X(N-1) = -X(N-2)$ . The received signal on subcarriers  $k$  and  $k+1$  to be written as

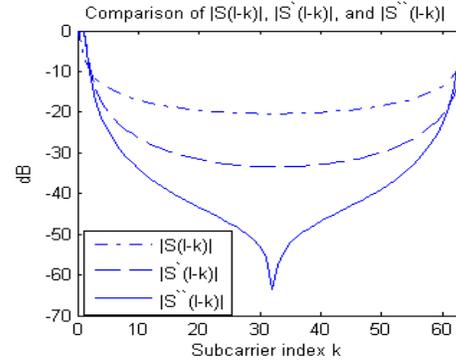
$$Y'(k) = \sum_{l=0,2,4,6,\dots}^{N-2} X(l)[S(l-k) - S(l+1-k)] + n_k \quad (4)$$

$$Y'(k+1) = \sum_{l=0,2,4,6,\dots}^{N-2} X(l)[S(l-k-1) - S(l-k)] + n_{k+1} \quad (5)$$

where  $n_k$  and  $n_{k+1}$  is the noise added to it.

And the ICI coefficient  $S'(l-k)$  is denoted as

$$S'(l-k) = S(l-k) - S(l+1-k) \quad (6)$$


**Figure 2: Comparison between  $|S'(l-k)|$ ,  $|S'(l-k)|$  and  $|S(l-k)|$** 

It is seen from figure 2 that  $|S'(l-k)| \ll |S(l-k)|$  for most of the  $l-k$  values. Hence, the ICI components are much smaller. Also, the total number of interference signals is halved in as opposed to since only the even subcarriers are involved in the summation.

#### 3.1.2 ICI Canceling Demodulation

ICI modulation introduces redundancy in the received signal since each pair of subcarriers transmit only one data symbol. This redundancy can be exploited to improve the system power performance, while it surely decreases the bandwidth efficiency.

To take advantage of this redundancy, the received signal at the  $(k+1)$ th subcarrier, where  $k$  is even, is subtracted from the  $k$ th subcarrier. This is expressed mathematically as

$$Y''(k) = Y'(k) - Y'(k+1) \\ = \sum_{l=0,2,4,\dots}^{N-2} X(l)[-S(l-k-1) + 2S(l-k) - S(l+k+1)] + n_k - n_{k+1} \quad (7)$$

Subsequently, the ICI coefficients for this received signal becomes

$$S''(l-k) = -S(l-k-1) + 2S(l-k) - S(l+k+1) \quad (8)$$

When compared to the two previous ICI coefficients  $|S(l-k)|$  for the standard OFDM system and  $|S'(l-k)|$  for the ICI canceling modulation,  $|S''(l-k)|$  has the smallest ICI coefficients, for the majority of  $l-k$  values, followed by  $|S'(l-k)|$  and  $|S(l-k)|$ . This is shown in Figure 2 for  $N=64$  and  $\varepsilon=0.5$ . The combined modulation and demodulation method is called the ICI self-cancellation scheme. The reduction of the ICI signal levels in the ICI self-cancellation scheme leads to a higher CIR. The theoretical CIR is given by

$$CIR = \frac{|-S(-1) + 2S(0) - S(1)|^2}{\sum_{l=2,4,6,\dots}^{N-1} |-S(l-1) + 2S(l) - S(l+1)|^2} \quad (9)$$

As mentioned previously, the redundancy in this scheme reduces the bandwidth efficiency by half. There is a tradeoff between bandwidth and power tradeoff in the ICI self-cancellation scheme.

#### 3.2 Maximum Likelihood Estimation

The second method for frequency offset correction in OFDM systems was suggested by Moose in [12]. In this approach, the frequency offset is first statistically estimated using a maximum likelihood algorithm and then cancelled at the receiver. This technique involves the replication of an OFDM symbol before transmission and comparison of the

phases of each of the subcarriers between the successive symbols.

When an OFDM symbol of sequence length  $N$  is replicated, the receiver receives, in the absence of noise, the  $2N$  point sequence i.e.,  $\{r(n)\}$  given by

$$r(n) = \frac{1}{N} \sum_{k=-K}^K X(k)H(k)e^{j2\pi n(k+\varepsilon)/N} \quad (10)$$

where  $\{X(k)\}$  are the  $2K+1$  complex modulation values used to modulate  $2K+1$  subcarriers,

The first set of  $N$  symbols are demodulated using an  $N$ -point FFT to yield the sequence  $R_1(k)$ , and the second set is demodulated using another  $N$ -point FFT to yield the sequence  $R_2(k)$ . The frequency offset is the phase difference between  $R_1(k)$  and  $R_2(k)$ , that is

$$R_2(k) = R_1(k) e^{j2\pi\varepsilon} \quad (11)$$

Adding the AWGN yields

$$\begin{aligned} Y_1(k) &= R_1(k) + W_1(k) \\ Y_2(k) &= R_1(k) e^{j2\pi\varepsilon} + W_2(k) \end{aligned} \quad (12)$$

$k = 0, 1 \dots N-1$

The maximum likelihood estimate of the normalized frequency offset is given by:

$$\hat{\varepsilon} = \frac{\text{Im} \sum_{k=-K}^K Y_2(k) Y_1^*(k)}{\text{Re} \sum_{k=-K}^K Y_2(k) Y_1^*(k)} \quad (13)$$

This maximum likelihood estimate is a conditionally unbiased estimate of the frequency offset and was computed using the received data. Once the frequency offset is known, the ICI distortion in the data symbols is reduced by multiplying the received symbols with a complex conjugate of the frequency shift and applying the FFT,

$$X(n) = \text{FFT} \{y(n) e^{j2\pi n\varepsilon/N}\} \quad (14)$$

### 3.3 Extended Kalman Filtering

A state space model of the discrete Kalman filter is defined as

$$z(n) = a(n) d(n) + v(n) \quad (15)$$

In this model, the observation  $z(n)$  has a linear relationship with the desired value  $d(n)$ . By using the discrete Kalman filter,  $d(n)$  can be recursively estimated based on the observation of  $z(n)$  and the updated estimation in each recursion is optimum in the minimum mean square sense.

The received symbols in OFDM System are

$$y(n) = x(n) e^{j2\pi n\varepsilon/N} + w(n) \quad (16)$$

where  $y(n)$  the received symbol and  $x(n)$  is the FFT of transmitted symbol. It is obvious that the observation  $y(n)$  is in a nonlinear relationship with the desired value  $\varepsilon(n)$ , i.e

$$\begin{aligned} y(n) &= f(\varepsilon(n)) + w(n) \\ \text{where } f(\varepsilon(n)) &= x(n) e^{j2\pi n\varepsilon/N} \end{aligned} \quad (17)$$

In order to estimate  $\varepsilon(n)$  efficiently in computation, we build an approximate linear relationship using the first-order Taylor's expansion:

$$y(n) \approx f(\hat{\varepsilon}(n-1)) + f'(\hat{\varepsilon}(n-1))[\varepsilon(n) - \hat{\varepsilon}(n-1)] + w(n) \quad (19)$$

$z(n)$  is linearly related to  $d(n)$ . Hence the normalized frequency offset  $\varepsilon(n)$  can be estimated in a recursive procedure similar to the discrete Kalman filter. As linear approximation is involved in the derivation, the filter is called the extended Kalman filter (EKF). The EKF provides a trajectory of estimation for  $\varepsilon(n)$ . The error in each update decreases and the estimate becomes closer to the ideal value during iterations.

## 4.2 ICI Cancellation

There are two stages in the EKF scheme to mitigate the ICI effect: the offset estimation scheme and the offset correction scheme.

### 4.2.1 Offset Estimation Scheme

To estimate the quantity  $\varepsilon(n)$  using an EKF in each OFDM frame, the state equation is built as

$$\varepsilon(n) = \varepsilon(n-1) \quad (23)$$

i.e., in this case we are estimating an unknown constant  $\varepsilon$ . This constant is distorted by a non-stationary process  $x(n)$ , an observation of which is the preamble symbols preceding the data symbols in the frame. The observation equation is

$$y(n) = x(n) e^{j2\pi n\varepsilon(n)/N} + w(n) \quad (24)$$

distorted in the channel,  $w(n)$  the AWGN, and  $x(n)$  the IFFT where  $y(n)$  denotes the received preamble symbols

of the preambles  $X(k)$  that are transmitted, which are known at

the receiver. Assume there are  $N_p$  preambles preceding the data symbols in each frame are used as a training sequence where  $\hat{\varepsilon}(n-1)$  is the estimate of  $\varepsilon(n-1)$ . To Define

$$z(n) = y(n) - f(\hat{\varepsilon}(n-1)) \quad (2)$$

$$d(n) = \varepsilon(n) - \hat{\varepsilon}(n-1) \quad (2)$$

$$1) \quad \text{and the following relationship}$$

$$z(n) = f'(\hat{\varepsilon}(n-1)) d(n) + w(n) \quad (2)$$

$$2) \quad \text{and the following relationship}$$

and the variance  $\sigma^2$  of the AWGN  $w(n)$  is stationary.

#### 4.2.2 Offset Correction Scheme

The ICI distortion in the data symbols  $x(n)$  that follow the training sequence can then be mitigated by multiplying the received data symbols  $y(n)$  with a complex conjugate of the estimated frequency offset and applying FFT, i.e.

$$\hat{x}(n) = \text{FFT}\{y(n) e^{-j2\pi n \hat{\epsilon}(n)/N}\} \quad (25)$$

As the estimation of the frequency offset by the EKF scheme is pretty efficient and accurate, it is expected that the performance will be mainly influenced by the variation of the AWGN.

#### 4.3 Algorithm

1. Initialize the estimate  $\hat{\epsilon}(0)$  and corresponding state error  $P(0)$
2. Compute the  $H(n)$ , the derivative of  $y(n)$  with respect to  $\epsilon(n)$  at  $\hat{\epsilon}(n-1)$  the estimate obtained in the previous iteration.
3. Compute the time-varying Kalman gain  $K(n)$  using the error variance  $p(n-1)$ ,  $H(n)$ , and  $\sigma^2$
4. Compute the estimate  $\hat{y}(n)$  using  $x(n)$  and  $\hat{\epsilon}(n-1)$  i.e. based on the observations up to time  $n-1$ , compute the error between the true observation  $y(n)$  and  $\hat{y}(n)$
5. Update the estimate  $\hat{\epsilon}(n)$  by adding the  $K(n)$ -weighted error between the observation  $y(n)$  and  $\hat{y}(n)$  to the previous estimate  $\hat{\epsilon}(n-1)$
6. Compute the state error  $P(n)$  with the Kalman gain  $K(n)$ ,  $H(n)$ , and the previous error  $P(n-1)$ .
7. If  $n$  is less than  $N$ , increment  $n$  by 1 and go to step 2; otherwise stop.

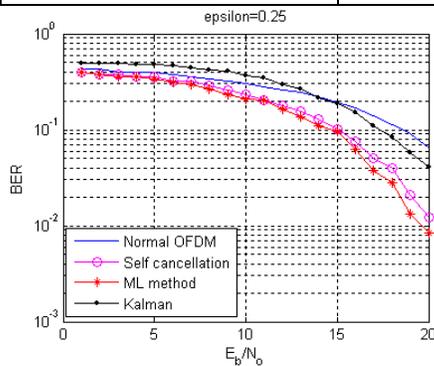
It is observed that the actual errors of the estimation  $\hat{\epsilon}(n)$  from the ideal value  $\epsilon(n)$  are computed in each step and are used for adjustment of estimation in the next step.

### 4. SIMULATIONS AND RESULTS

In order to compare the ICI cancellation schemes, BER curves were used to evaluate the performance of each scheme. For the simulations in this project, MATLAB was employed. The simulations were performed using an AWGN channel.

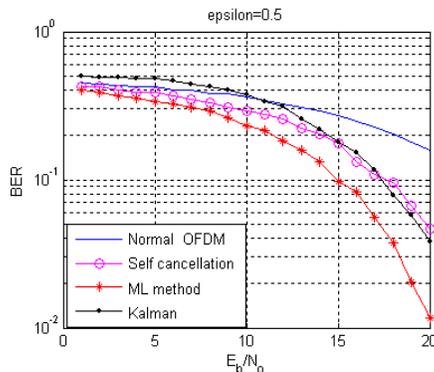
**Table 1: Simulation Parameters**

PARAMETERS	VALUES
Number of carriers (N)	1705
Modulation (M)	BPSK
Frequency offset $\epsilon$	[0.25,0.5,0.75]
No. of OFDM symbols	100
Bits per OFDM symbol	$N \cdot \log_2(M)$
$E_b$ - $N_0$	1:20
IFFT size	2048



**Figure 3: BER performance with ICI Cancellation for  $\epsilon=0.25$**

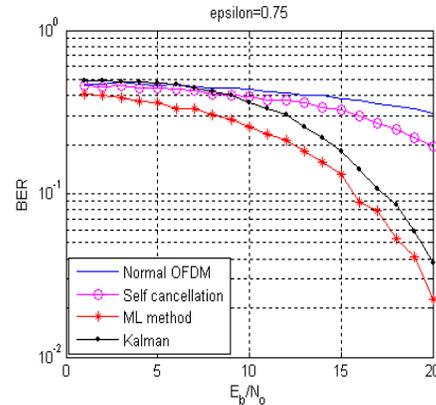
Figure 3 shows that for small frequency offset values, ML and SC methods have a similar performance. However, ML method has a lower bit error rate for increasing values of  $E_b/N_0$ .



**Figure 4: BER performance with ICI Cancellation for  $\epsilon=0.5$**

Figure 4 illustrates that for frequency offset value of 0.5, BER increases for both the methods but ML method maintains a lower bit error rate than SC. EKF is better than SC method.

In figure 5, for frequency offset value of 0.75, self-cancellation method has a BER similar to standard OFDM system since the self-cancellation technique does not completely cancel the ICI from adjacent sub-carriers and



**Figure 5: BER performance with ICI Cancellation for  $\epsilon=0.75$**

the effect of this residual ICI increases for larger offset values. However, ML method has an increased BER performance and proves to be efficient than SC method.

### 5. CONCLUSION

It is observed from the figures that Extended Kalman filter method indicates that for very small frequency offset, it does not perform very well, as it hardly improves BER. However, for high frequency offset the Kalman filter does perform extremely well. Important advantage of EKF method is that it does not reduce bandwidth efficiency as in self cancellation method because the frequency offset can be estimated from the preamble of the data sequence in each OFDM frame.

Self cancellation does not require very complex hardware or software for implementation. However, it is not bandwidth efficient as there is a redundancy of 2 for each carrier. The ML method also introduces the same level of redundancy but provides better BER performance, since it accurately estimates the frequency offset. EKF implementation is more complex than the ML method but provides better BER performance.

Further work can be done by extending the concept of self-ICI cancellation and by performing simulations to investigate the performance of these ICI cancellation schemes in multipath fading channels.

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